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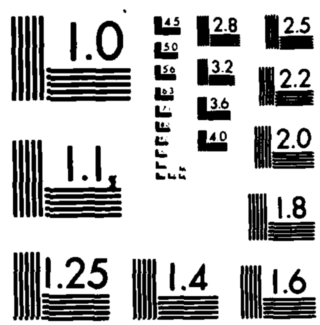

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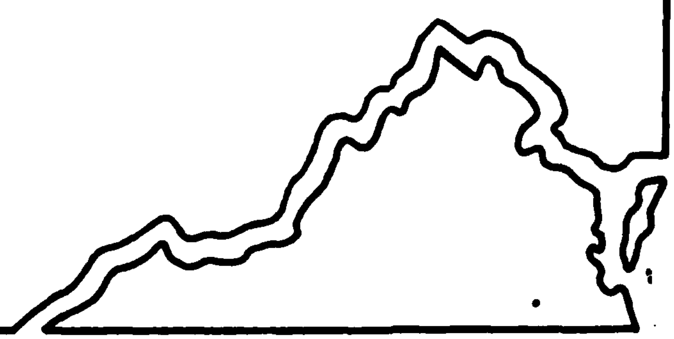

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The authors consider the random-accessing problem of a single, collision-type, slotted, packet-switched communication channel by a large number of independent, data transmitting bursty users. The authors propose and analyze an easy-to-implement algorithm under the realistic assumption that each user inspects the channel outcome feedback only whenever he is blocked. The authors assume binary feedback which informs the users only about whether or not there was a collision in the previous slot. The authors show that the algorithm results in finite average delays for transmission at (CONTINUED)

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ITEM #20, CONTINUED: rates less than 0.36 packets per channel slot, and the authors give an exact upper bound for the average delay.

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LIMITED SENSING RANDOM MULTIPLE ACCESS

USING BINARY FEEDBACK

by

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Abstract

The authors ~~We~~ consider the random-accessing problem of a single, collision-type, slotted, packet-switched communication channel by a large number of independent, data transmitting bursty users. They ~~We~~ propose and analyze an easy-to-implement algorithm under the realistic assumption that each user inspects the channel outcome feedback only whenever he is blocked. ~~We assume~~ Assumed is binary feedback which informs the users only about whether or not there was a collision in the previous slot. It is ~~We~~ ~~show~~ shown that the algorithm results in finite average delays for transmission at rates less than 0.36 packets per channel slot, and we give an exact upper bound for the average delay is given.

1. Introduction

The multiple-access problem in communications is the problem of organization, or coordination of a population of users for the efficient sharing of the resources of a single channel used by the users for information transmission. This situation arises in a number of applications: Computer-communication networks, packet-radio networks, satellite communication networks, local area networks.

Random multiple-access schemes are an important class of techniques that employ distributed control algorithms to cope with the multiple access problem. These schemes are especially useful in the presence of an asymptotically large number of ill-specified, independent bursty users. The users gain access into the channel on a contention basis. The accessed channel is a collision-type, packet switched, time-slotted transmission channel. Some form of feedback information associated with the message transmissions is always assumed to be available to the contenting users. For this general model, a variety of access algorithms (protocols) has been proposed and analyzed by several authors. The properties of these algorithms vary considerably with the level of the feedback information assumed available to the users.

In [4-8] it is assumed that immediately after each channel slot a ternary feedback is broadcasted to the users. This is

known as 0,1,e feedback and informs the users whether the previous slot was empty (0), or contained one packet (1), or contained a collision (e). A collision occurs whenever more than one users attempt transmission within the same slot. All information contained in the collided packets is assumed lost, and these packets must be retransmitted at later times.

The algorithms developed in [9] use binary feedback. Binary feedback is less informative compared to ternary feedback and may be available in three different forms: "Collision/No Collision" feedback, "Something/Nothing" feedback and "Success/Failure" feedback (notation suggested by Mehravari and Berger [9]).

Several recent efforts of developing more efficient realizable algorithms have used more informative types of feedback than ternary feedback. In [10] it is assumed that after each collision the number (up to an upper maximum limit) of the packets involved is revealed to all users, through a bank of energy detectors. Also, in [11] it is assumed that additional information (four-valued, or five-valued feedback) is available to the users through the use of control mini-slots. All the algorithms in the papers mentioned so far require that each user inspects the feedback broadcasting for every channel slot over the entire operation of the random-access system. In the slotted Aloha algorithm [1, 2] (which is unstable for an asymptotically large number of users) each user inspects only the slots that correspond to his own attempts.

Tsybakov and Vvedenskaya [12] proposed and analyzed a "limited-sensing" algorithm, called "Stack" algorithm, where

each user inspects the feedback broadcasting only whenever there is an unsuccessfully transmitted packet in his buffer. The "Stack" algorithm uses ternary feedback and achieves a maximum stable throughput of at least 0.384 packets per slot.

Using the same feedback level as in [12] Papantoni-Kazakos and Marcus [13] developed a limited channel sensing algorithm for a limited number of data users.

In the present paper, we propose and analyze a Limited Sensing random access Algorithm with Binary Feedback (LSBFA). The user and channel models assumed are described in section 2. In the LSBFA, users with new packets transmit their packets in the first slot following their arrival, and then they resolve any collisions using the "Capetanakis-Tsybakov-Mikhailov-Collision-Resolution-Algorithm" (CTMCRA) [14]. In contrast to the "Tree-type" random access algorithms [4-11], where there is an explicit and separate collision resolution period, the LSBFA, like the "Stack" algorithm, allows new packets to continuously enter into the system independently of the collision resolution process already in progress. From a practical point of view, the "continuous-entry" feature is very significant, since the users monitor the feedback channel <u>only</u> whenever they have a packet to send (limited-sensing). Thus, the undesirable in several applications necessity of all users monitoring the feedback channel constantly--even if they have no packet to send--is eliminated.

The organization of the paper is as follows:

In section 2, we present the user and channel models, and the LSBFA statement and general operation.

In section 3, we analyze the algorithm by expressing and studying a system of recursive equations for the expected length of collision resolution sessions; we evaluate the stability region of the algorithm and the expected length of a session.

In section 4, we use the results of section 3 to give an exact upper bound for the average packet delay.

In section 5, we compare the LSBFA to other random access algorithms.

2. The Model and the Statement of the Algorithm

We assume that an infinite population of spatially isolated, independent, bursty users (transmitters) share a single channel to communicate with a central facility (common receiver). The users transmit data packets of fixed duration taken to be the unit of time. The channel time is divided into unit-time-segments called slots. The unit interval $(t, t+1)$ is called slot t $(t = 0, 1, 2, \ldots)$. All users are synchronized to the starting points of the channel slots, and they attempt transmission of some packet only at the beginning of some channel slot.

The users do not communicate with each other directly; therefore, the probability of more than one users attempting transmission of a packet within the same slot is nonzero. A channel slot is a collision slot if more than one packets attempted transmission within it. All information in the packets involved in a collision is assumed lost, and these packets have to be retransmitted. A channel slot is empty if no packet attempted transmission within it, and it is successful if exactly one packet was transmitted within it. In the later case it is assumed that the transmitted packet reaches its destination error free.

A feedback channel from the common receiver informs the transmitters at the end of each slot whether or not there was a collision in that slot. We assume that the feedback channel

is a noiseless broadcast channel, and that propagation delays are negligible. Therefore, immediately at the end of slot t a user who is interested in the outcome of that slot can learn it be monitoring the feedback channel. Let the random variable Z_t denote the outcome of slot t. We have

$$Z_t = \begin{cases} \text{NC if slot t contained} \leq 1 \text{ packets (no collision)} \\ \text{C if slot t contained} \geq 2 \text{ packets (collision)} \end{cases}$$

The Collision/No Collision (CNC) binary feedback assumed here uses less information compared to the 0,1,e ternary feedback, since the former does not distinguish an empty slot from a successful slot. Its implementation can be based on a simple binary acknowledgment scheme from the central facility to the users ("NC" or "C").

Let the random variable N(t) denote the number of new packets appearing in the system for transmission from all users combined during slot t. It is assumed that $\{N(t)\}(t = 0, 1, 2, \ldots)$ is a sequence of independent and identically distributed random variables. Let $p_n = \Pr\{N(t) = n\}$ be the probability mass function (p.m.f.) of $N(t)(t = 0, 1, 2, \ldots)$, and let $\lambda = E\{N(t)\}$ be its expectation, which is assumed being finite. Thus, λ is the intensity of the cumulative input traffic measured in number of packets per slot. For the infinite-population model, only one packet requiring transmission can be present at a station at any given point of time.

The following definition will be used in the statement of the algorithm.

Definition 1 At any time t a user may be either active or inactive

At any time t an active user may be either new or blocked

A new user at time t is a user with a packet generated during slot t-1

A blocked user at time t is a user with a packet that has attempted transmission and experienced collision at some slot prior to slot t

At time t a packet is new or blocked if it belongs to a new or blocked user respectively

The statement of the algorithm is contained in two simple rules. For the implementation of the algorithm in a distributed fashion, it suffices for each user to have a counter and a binary fair coin. The rules of the algorithm are followed by the active users only, and are as follows:

Let the random-access system start at t=0 with all counters set at 0.

Rule 1 At time t (t = 0, 1, 2 . . .) blocked users with counter at 0 and all new users transmit their packets in slot t

Rule 2 At the end of slot t (just prior to time t+1) all active users inspect the feedback channel. If slot t were collison free (Z_t = NC), the user who transmitted his packet (if any) leaves the system (becomes inactive), and all blocked users decrement their counters by one. If slot t were a collision slot (Z_t = C), each collided user tosses a binary

fair coin and sets his counter to 0 or to 1 according to the outcome of the coin tossing. All other blocked users increment their counters by one.

3. Algorithm Analysis

The two most important performance measures of a random-access algorithm are the average delay of a packet and the maximum stable throughput of the system. The delay of a packet is the time between the instant the packet originates as a new packet until the instant it is successfully transmitted. Let δ_n be the random variable that denotes the delay of the n^{th} packet.

A random-access algorithm or system is called stable if the $\limsup_{n\to\infty} E\{\delta_n\}$ is finite, assuming that the limit exists. This means that for a stable algorithm the delay of a packet will remain finite with probability one. The throughput or output rate of a random-access system is the long-run average number of successfully transmitted packets per unit time; it is denoted by η. Given an algorithm, let η_* be the supremum of η. If the input rate λ is less than η_*, then the throughput is λ and the system is stable. If the input rate λ exceeds the maximum output rate η_*, then the average packet delay becomes unbounded (Little's result) and the system is unstable. Thus,

$$\eta_* = \sup\{\lambda: \text{ the system is stable}\}$$

We call η_* the efficiency of the given algorithm. The interval $(0, \eta_*)$ is its stability region.

In this section we study the stability region and the average packet delay for the algorithm described in section 2. We proceed with the following definitions:

Definition 2 A clock instant t_R is a renewal instant if all active users at t_R (if any) are new users

Definition 3 A session is a sequence of consecutive channel slots, that begins and ends at two consecutive renewal instants.

Definition 4 A session starting at some renewal instant t_R is called a session of multiplicity k if the number of active (new) users at t_R is k; $k = 0, 1, 2, \ldots$

If t_R is a renewal instant, then at t_R, by definition 2, there are no blocked users in the system. Thus, any previously blocked packet has been successfully transmitted prior to slot t_R. From definitions 1 and 3, and the independence of the incoming traffic from a particular session, it is clear that the lengths of consecutive sessions are independent and identically distributed random variables. Let the random variable τ denote the length of an arbitrary session, and let τ_k denote the length of a session of multiplicity k. The distribution of both random variables τ and τ_k depends only on the probability mass function, that models the incoming traffic (input), and on the rules of the algorithm, but not on the particular session.

The sessions with multiplicity 0 or 1 are trivial and both have length equal to one slot. The session of multiplicity 0 is called the empty session.

Let t_s and t_e be the two random consecutive renewal instants that denote the starting and ending instants of the n^{th} nonempty session respectively. Since at both t_s and t_e there are no blocked users present at the system, the number of successfully transmitted packets during the course of the session is simply the random number of packets that appeared to the system requiring transmission during the time interval $(t_s-1,\ t_e-1)$. Let the random variable M denote the random number of successfully transmitted packets during the nth nonempty session. The average packet delay of a nonempty session is defined as follows:

$$D = E\{M^{-1} \sum_{m=1}^{M} \delta_m\} \tag{1}$$

where δ_m is the delay experienced by the mth successfully transmitted packet during the session.

It is clear that D is independent of the particular session, because sessions are independent of each other, and because the input traffic is a process with independent and identically distributed increments (N(t)). Thus, the average packet delay of the n^{th} session, given by (1), is the average packet delay of the system, that is over all sessions.

Since all the packets of the session requested transmission not earlier than t_s, and were successfully transmitted not later than t_e-1, we have

$$\delta_m \leq \tau-1; \quad m = 1, 2, \ldots \tag{2}$$

where τ is the length of the session. From (1) and (2) we have

the following upper bound for D:

$$D \leq L - 1 \tag{3}$$

where $L = E\{\tau\}$ is the mean length of a nonempty session. Let L_k be the mean length of a session of multiplicity k. Then, $L_k = E\{\tau_k\}$. The mean length L of a nonempty session is expressed in terms of L_k and the probability mass function p_k of the input, as follows:

$$L = (1-p_0)^{-1} \sum_{k=1}^{\infty} p_k L_k \tag{4}$$

The algorithm is stable if and only if D is finite. Hence, using the bound given by (3), the condition $L<\infty$ is sufficient for stability. Furthermore, the region of convergence of the series given by (4) is a subset of the stability region of the algorithm.

We proceed now with the investigation of the region of convergence of the series given by (4) by deriving and studying a system of equations for L_k; $k = 0, 1, 2, \ldots$

3.1 Mean Length of a Session of Given Multiplicity

Theorem 1

Let τ_k be the random length of a session of multiplicity k. Then,

$$\tau_0 = \tau_1 = 1$$

and

$$\tau_k = 1 + \tau_{I+M} + \tau_{k-I+N}; \quad k \geq 2 \tag{5}$$

where I, M, N are independent random variables.

The random variable I is binomially distributed:

$$\Pr\{I=i\} = b_k(1) \triangleq \binom{k}{i} 2^{-k}$$

The random variables M and N are identically distributed with

$$\Pr\{M=i\} = \Pr\{N=i\} = p(i)$$

where $p(\cdot)$ is the p.m.f. of the input increment.

<u>Proof</u>

For the trivial sessions of multiplicity 0 or 1, by definition, $\tau_0 = \tau_1 = 1$. For $k \geq 2$ let the session start at the renewal instant t_s with k new users. According to the first rule of the algorithm all the k users transmit their packets in slot t_s. Hence, slot t_s is always a collision slot. At instant t_s+1 there are no other blocked users except for the k collided users, who, according to the second rule, toss a fair coin and set their counters either to 0 or to 1. Let the random variable I denote the number of those from the k users, who set their counters to 0. Then, k-I is the random number of blocked users with their counters set at 1. Clearly, $\Pr\{I=i\} = \binom{k}{i} 2^{-k}$; $i = 0, 1, \ldots, k$.

Let the random variable M denote the number of new users at instant t_s+1. These are the users with a packet originated during slot t_s. Clearly, $\Pr\{M=m\} = \Pr\{N(t_s) = m\} = p_m$, and M is independent of I. According to the algorithm a total number of I+M users transmit their packets in slot t_s+1, while the k-I blocked users with their counters previously set to 1 inspect the feedback channel and increment (decrement) their counters by one for each subsequent collision (collision-free) slot respectively.

The crucial observation here is that all the $k-I$ blocked users have identical counter indication until the random instant t_o at which this indication becomes 0 for the first time. Furthermore, it is not difficult to see from the rules of the algorithm, that the identical counter indication of the $k-I$ blocked users is always greater than the counter indication of any other blocked user in the system. Thus, at instant t_o there are no other blocked users in the system except of the $k-I$ users who all have their counters set to 0. This means that the $I+M$ packets that started accessing the channel in slot t_{s+1} and all the packets that appeared to the system during (t_s-1, t_o-1) have been successfully transmitted by the random instant t_o.

Let the random variable N denote the number of new users at t_o. Clearly, $Pr\{N=n\} = Pr\{N(t_o-1) = n\} = p_n$ and N is independent of I and M. In slot t_o a total of $k-I+N$ users transmit their packets, since there are $k-I$ blocked users with counter indication 0 and N new users. Consequently, the $I+M$ users may be thought of as starting an independent session of random multiplicity $I+M$, that begins at t_s+1 and ends at t_o. This session is immediately followed by a session of multiplicity $k-I+N$. This later session starts at t_o and ends at t_e, which is the first renewal instant after t_s. Thus,

$$\tau_k \triangleq t_e-t_s = 1 + [t_o - (t_s+1)] - (t_e - t_o)$$

$$= 1 + \tau_{I+M} + \tau_{k-I+N} \qquad \text{Q.E.D.}$$

Let $G(k, z)$, $0 \leq z \leq 1$ denote the moment generating function of the random variable $\tau_k - 1$:

$$G(k, z) = E(z^{\tau_k - 1})$$

In view of Theorem 1, the following theorem is straightforward:

Theorem 2

$$G(0, z) = G(1, z) = 1$$

$$G(k, z) = \sum_{i=0}^{k} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} b_k(i)p(m)p(n)z^2 \, G(i+m, z)G(k-i+n, z) \tag{6}$$

where $b_k(i) = \binom{k}{i}2^{-k}$ and $p(\cdot)$ is the p.m.f. of the input increment.

For the mean length of a session of multiplicity k we have:

$$L_k = E(\tau_k) = \frac{\partial}{\partial z} G(k, z)\Big|_{z=1} + 1; \quad k = 0, 1, 2, \ldots$$

By differentiating (6) we respect to z, or by directly taking expectations in (5) we have:

Corollary 1

$$L_0 = L_1 = 1$$

$$L_k = 1 + 2 \sum_{i=0}^{k} \sum_{j=0}^{k} b_k(i)p(j)L_{i+j} = 1 + 2 \sum_{m=0}^{\infty} q_k(m)L_m; \; k \geq 2 \tag{7}$$

where

$$q_k(m) = b_k(m) * p(m) = \sum_{i=0}^{v} b_k(i)p(m-i) \tag{8}$$

$$v = \min(k, m)$$

---

*denotes convolution

It is noteworthy that the coefficients $q_k(\cdot)$, given by (8), are the values of the convolution of the binomial p.m.f. $b_k(\cdot)$ with the p.m.f. of the input increments $p(\cdot)$. Thus, $q_k(\cdot)$ is the p.m.f. of the sum I+J of two independent random variables: The random variable I, which is binomially distributed, and the random variable J, which is distributed according to $p(\cdot)$. But this is exactly what the continuous entry algorithm does to resolve collisions. After a collision the collided users (k) toss a fair coin, and those who tossed 0 (I) transmit their packets in the next slot along with the newcomers (J). Note also that for $p(0) = 1$, $p(i) = 0$, $i > 0$ equation (7) becomes equation (3.12) of [14], that gives the system of equations satisfied by the conditional mean length of a collision resolution interval for the CTMCRA.

The system of linear equations for L_k, given by (7), will be of central interest in this paper. In what follows, we investigate the conditions, under which system (7) has a unique nonnegative solution, such that $0 \leq L_k < \infty$ for $0 \leq k < \infty$.

3.2 Stability

We consider a general system in $_k$ that corresponds to the system given by (7):

$$x_0 = x_1 = 1$$

$$x_k = 1 + 2 \sum_{m=0}^{\infty} q_k(m) x_m; \quad k \geq 2 \tag{9}$$

where $q_k(m)$ are as given in (8).

We are interested in investigating the conditions, under which system (9) has a nonnegative solution, that is bounded for finite k.

Given a sequence $Y = \{y_k\}$ with $y_k \varepsilon R$ $(k = 0, 1, 2, \ldots)$, we define the operators A and B (assuming that they exist) as follows:

$$A[Y] = \{A_k[Y]\}; \quad k = 0, 1, 2, \ldots$$

such as: $A_0[Y] = A_1[Y] \triangleq 1$

$$A_k[Y] \triangleq 1 + 2 \sum_{m=0}^{\infty} q_k(m) y_m; \quad k \geq 2 \tag{10}$$

and

$$B[Y] = \{B_k[Y]\}; \quad k = 0, 1, 2, \ldots$$

such as: $B_0[Y] = B_1[Y] \triangleq 0$

$$B_k[Y] \triangleq 2 \sum_{m=2}^{\infty} q_k(m) y_m; \quad k \geq 2 \tag{11}$$

Let also $A^n[Y](B^n[Y])$ denote the sequence resulting from the n times repeated application of operator A(B) respectively on the initial sequence Y $(A^\circ[Y] = B^\circ[Y] = Y)$.

Using the operator A system (9) becomes:

$$x_0 = x_1 = 1$$
$$x_k = A_k[X]; \quad k \geq 2$$

The following theorem gives a general sufficient condition, under which system (9) has a unique, nonnegative, bounded (for finite k) solution.

<u>Theorem 3</u>

In the class of sequences X such that

$$\lim_{i\to\infty} \max_{k<i} \sum_{m=i}^{\infty} q_k(m)\,|x_m| = 0 \tag{12}$$

system (9) has a unique solution $X = \{x_k\}$ if the p.m.f. of the input increment $p(\cdot)$ is such that, there exists some $n_0 < \infty$ ($n_0 = 1, 2, \ldots$), such that

$$(B^{n_0})_k[F(\lambda, r)] > 0 \quad \text{for every } k \geq 2 \tag{13}$$

where

$$F(\lambda, r) = \{2^{-k} f_k(\lambda, r)\}; \quad k \geq 2$$

$$f_k(\lambda, r) = (1 - 2\lambda)k + r(1 - 2\lambda) - 2\lambda \tag{14}$$

$\lambda = \sum_{i=0}^{\infty} ip(i)$, $r = p(1)/p(0)$, and B is the operator defined in (11)

If (13) is true, then for every k, we have:

$$0 \leq (A^n)_k[{}^{(0)}X] \leq x_k \quad \text{for every } n \geq 0;$$

$$x_k \le (A^n)_k[x^{(0)}] \text{ for every } n \ge n_0, \text{ and}$$

$$x_k = \lim_{n\to\infty}(A^n)_k[{}^{(0)}X] = \lim_{n\to\infty}(A^n)_k[X^{(0)}]$$

where

$${}^{(0)}X = \{{}^{(0)}{}_k\} \text{ with } {}^{(0)}x_0 = {}^{(0)}x_1 = 1 \text{ and } {}^{(0)}x_k = b'k-c';$$

$$k \ge 2$$

$$b' = 2/(1-2\lambda),\ c' = 2b'\lambda+1$$

$$X^{(0)} = \{x_k{}^{(0)}\} \text{ with } x_0^{(0)} = x_1^{(0)} = 1 \text{ and } x_k^{(0)} = bk-c;\ k \ge 2$$

$$b = \max_{k\ge 2}((B^{n_0})_k[G(r)]/(B^{n_0})_k[F(\lambda,\ r)])$$

$$G(r) = \{2^{-k}g_k(r)\},\quad g_k(r) = 2(k+r+1)$$

$F(\lambda,\ r)$ is as defined in (14), A is as defined in (10)

and B is as defined in (11)

The proof of this theorem is given in Appendix A.

For $n_0 = 0$, condition (13) of Theorem 3 becomes:

$$(B^0)_k[F(\lambda,r)] = 2^{-k}f_k(\lambda,r) > 0 \quad \text{for every } k \ge 2$$

The above condition is satisfied if $f_2(\lambda,r) > 0$, since then $f_k(\lambda,r)$ is a monotone increasing sequence.

<u>Corollary 2</u>

In the class of sequences X, satisfying (12), system (9) has a unique solution $X = \{x_k\}$,

$$0 < b'k - c' \leq {}_{k} \leq bk - c$$

if $\quad f_2(\lambda, r) = (1 - 2\lambda)(r + 2) - 2\lambda > 0 \qquad (15)$

where

$$b = g_2(r)/f_2(\lambda, r) = 2(r + 3)/((1 - 2\lambda)(r + 2) - 2\lambda)$$

and

λ, r, b', c', c are as defined in Theorem 3.

If the incoming traffic is Poisson distributed, that is

$$p(i) = \frac{\lambda^i}{i!} e^{-\lambda}$$

then, condition (15) can be expressed in terms of the traffic intensity λ only, since $r = P(1)/P(0) = \lambda$.

<u>Corollary 3</u>

For the Poisson distribution, system (9) has a unique solution $X = \{x_k\}$ satisfying (9),

$$\frac{2}{1-2\lambda}(k-2\lambda) - 1 \leq x_k \leq \frac{2(\lambda+3)}{\lambda+2-2\lambda(\lambda+3)}(k-2\lambda) - 1$$

for $\lambda < \lambda_p = 0.35078$

where λ_p is the positive root of the equation $\lambda+2-2\lambda(\lambda+3) = 0$.

For the rest of the paper, we assume that the input process is Poisson. The following lemma gives an easily computable recursive expression of the n^{th} power of the operator B, that will be very helpful in improving the sufficient condition given in Corollary 3, and in calculating the solution of system (9) (if it exists):

Lemma 1

Let the sequence $Y = \{y_k\}$ be such that

$$y_k = (a_0 + b_0 k)2^{-k}; \quad k \geq 2$$

where

$$a_0,\ b_0 \varepsilon R$$

Then, for the Poisson distribution

$$(B^n)_k[Y] = \sum_{i=0}^{n} (a_n(i) + kb_n(i))v_i^k \tag{16}$$

where

$$v_0 = 1/2, \quad v_{i+1} = (1+v_i)/2, \quad a_0(0) = a_0,\ b_0(0) = b_0$$

$$a_n(0) = -2\exp(-\lambda) \sum_{i=0}^{n-1} ((1+\lambda v_i)a_{n-1}(i) + \lambda v_i b_{n-1}(i))$$

$$b_n(0) = -2\exp(-\lambda) \sum_{i=0}^{n-1} (a_{n-1}(i) + b_{n-1}(i))v_i$$

$$a_n(i) = 2\exp(-\lambda(1-v_{i-1}))(a_{n-1}(i-1) + \lambda v_{i-1}\, b_{n-1}(i-n)), \quad i \leq i \leq n$$

$$b_n(i) = 2\exp(-\lambda(1-v_{i-1}))v_{i-1}(1 + v_{i-1})^{-1}\, b_{n-1}(i-1), \quad 1 \leq i \leq n$$

The proof of Lemma 1 is given in Appendix B.

Theorem 4

For p(i) the Poisson distribution, problem (9) has a unique nonnegative solution satisfying (12) if

$$\lambda < 0.3601$$

Proof

Using condition (13) of theorem 3, it suffices to find some $n_0 < \infty$ such that:

$$(B^{n_0})_k[F(\lambda)] > 0 \quad \text{for every } k \geq 2 \text{ and } \lambda < \bar{\lambda} = .3601$$

where

$$F(\lambda) = \{2^{-k} f_k(\lambda)\}$$

$$f_k(\lambda) = (1-2\lambda)k - \lambda(1+2\lambda); \quad k \geq 2$$

If $\lambda_1 < \lambda_2$ then $(B^{n_0})_k[F(\lambda_1)] \geq (B^{n_0})_k[F(\lambda_2)]$, since, for every $k \geq 2$, $f_k(\lambda)$ is a monotone decreasing function of λ, and the defining coefficients $q_k(m)$ of the operator B in (11) are nonnegative. Thus, to prove the theorem, it suffices to find some $n_0 < \infty$, such that:

$$(B^{n_0})_k[F(\bar{\lambda})] > 0 \quad \text{for every } k \geq 2 \tag{17}$$

Let $a_0 = -\bar{\lambda}(1 + 2\bar{\lambda})$ and $b_0 = (1 - 2\bar{\lambda})$. Then, using Lemma 1 it is not difficult to show that (17) is true for $n_0 \geq 5$.

To indicate the tightness of the above sufficient condition, we mention that $(B^{n_0})[F(0.3602)]$ remains negative for at least $n_0 \leq 75$.

The next two theorems are parallel to Theorem 4 and 5 in [12].

Theorem 5

For $p(i)$ the Poisson distribution, system (9) has no nonnegative solution satisfying (12) if

$$\lambda > 0.363.$$

The proof of Theorem 5 can be found in Appendix C.

Theorem 6

If system (9) has a solution satisfying (12), then it increases not more rapidly than linearly with k.

The proof of Theorem 6 is omitted.

We proceed with a theorem that links the finite solution of system (9) (if it exists) to the solution $\{L_k\}$ of system (7) for the mean session length with specified multiplicity of the algorithm. This is necessary, since, formally, system (9) always has the trivial solution $x_0 = x_1 = 1$, $x_k = \infty$; $k \geq 2$. We have

Theorem 7

If $X = \{x_k\}$ is a solution of system (9) satisfying condition (12), then $L_k = x_k$, $k \geq 0$.

The above theorem is parallel to Theorem 6 in [12] and its proof is omitted.

In view of Theorems 4, 6, and 7 we conclude that the average session length L given by (4) is finite if $\lambda < .3601$. But if L is finite, then the average delay D is also finite, since $D \leq L-1$. Hence, for the Poisson distribution the algorithm is stable if the input intensity λ is less than 0.3601 packets per channel slot.

4. Average Delay

In this section, we calculate the mean session length L, which serves as an upper bound for the average packet delay D in the stable region of the algorithm.

We first solve system (7) to find the mean session length of specified multiplicity L_k for λ in the stable region. From Theorem 3 we have

$$(A^n)_k[{}^{(0)}X] \leq L_k \leq (A^n)_k[X^{(0)}] \quad \text{for every } n \geq n_0; \quad k \geq 2$$

and

$$L_k = \lim_{n\to\infty}(A^n)_k[{}^{(0)}X] = \lim_{n\to\infty}(A^n)_k[X^{(0)}]; \quad k \geq 2$$

where A, n_o, ${}^{(0)}X$, $X^{(0)}$ are as defined in Theorem 3.

After simple calculations, we have

$$A_k[{}^{(0)}X] = b'k-c' + 2p(0)b'2^{-k}; \quad k \geq 2$$

$$A_k[X^{(0)}] = bk-c - 2p(0)((b(1-2\lambda)-2\lambda)k-\lambda(1+2\lambda)b-2(\lambda+1)2^{-k}; \quad k \geq 2$$

where ${}^{(0)}X$, $X^{(0)}$, b, b', c, c' are as defined in Theorem 3.

In view of (10) and (11) it is not difficult to prove that the following are true:

$$(A^n)_k[{}^{(0)}X] = b'k-c' + \sum_{i=0}^{n-1} (B^i)_k[{}^{(0)}Y] \tag{18}$$

$$(A^n)_k[X^{(0)}] = bk-c - \sum_{i=0}^{n-1} (B^i)_k[Y^{(0)}] \tag{19}$$

where

$$^{(0)}Y = \{^{(0)}y_k\}; \quad ^{(0)}y_k = a_0' \, 2^{-k}; \quad k \geq 2$$

$$a'_0 = 2p(0)b'$$

and

$$Y^{(0)} = \{y_k^{(0)}\}; \quad y_k^{(0)} = (a_0 + b_0 k)2^{-k}; \quad k \geq 2$$

$$a_0 = -2p(0)(\lambda(1+2\lambda)b-2(\lambda+1))$$

$$b_0 = 2p(0)(b(1-2\lambda)-2\lambda)$$

The forms of the lower and upper bound for the solution L_k given in (18) and (19) respectively are well suited for the application of Lemma 1. We used Lemma 1 to calculate the powers of the operator B appearing in (18) and (19). For $\lambda \leq 0.3$ we found that the values of the upper and lower bound coincide up to the fourth decimal point within the first fifty iterations. We should note here that using Lemma 1 one can calculate the exact solution of system (7) for the mean session length with arbitrary accuracy for any λ in the region of stability of the algorithm.

In Table 1, we give the values of the mean length of sessions of multiplicities up to ten for different values of the input intensity.

The values of L_k given in Table 1 were used in (4) to calculate the mean session length L. The results are plotted in Figure 1, which gives the upper bound L-1 for the average packet delay of the algorithm

5. Comparison of the LSBFA to Other Access Algorithms

A random-access algorithm that can be implemented using CNC binary feedback was first treated by Capetanakis [4, 5], and by Tsybakov and Michailov [6]. This algorithm uses the CTMCRA to resolve collisions and there are two versions of it, a static version and a dynamic version. For the Poisson model and infinite user population the static algorithm achieves a maximum stable throughput of 0.346 packets per slot (p.p.s.), while the dynamic algorithm achieves a maximum stable throughput of 0.429 packets per slot. Recently, Mehravari and Berger [9] proposed a first-come-first-served collision resolution algorithm with CNC binary feedback, which is stable for input rates less than 0.4422 packets per slot. Also, Hajek and Van Loon [15] have recently shown that Aloha-type retransmission control policies, that achieve a maximum stable throughput of $e^{-1} = 0.3678$ packets per slot, can be implemented on a random-access channel using CNC binary feedback.

All these schemes assume that each user monitors the feedback channel constantly (for every channel slot at all times) even if he has no packet to send. This feedback requirement can be reduced in the scheme proposed in [15] at the expense of increased average delay.

In contrast to all the schemes mentioned above, the LSBFA eliminates completely this undesirable and not practical

necessity by allowing continuous entry of new packets into the random-access system. Furthermore, the LSBFA is easier to implement than all these schemes--its implementation requires only a single counter possessed by each user.

In section 3 we showed that the LSBFA is stable for input rates less than 0.3601 packets per slot. Obviously, as the level of feedback information inspected by each user decreases, the maximum stable throughput decreases also. Notice, however, that the LSBFA outerperforms the static "Tree" algorithm [4, 5] (0.360 p.p.s. versus 0.346 p.p.s.), even though the later uses more feedback information.

The LSBFA uses the "continuous-entry" idea introduced in the "Stack" algorithm [12]. The "Stack" algorithm uses ternary feedback (0,1,or e),and for the Poisson-infinite-population model it is stable for input rates less than 0.384 packets per slot. Thus, in cases where a central facility supplies the feedback information by an Acknowledgment scheme, simplifying from ternary feedback to CNC binary feedback does not significantly reduce the efficiency.

The LSBFA is easier to implement than the "Stack" algorithm because it eliminates the memory necessity in the later algorithm. In the "Stack" algorithm each blocked user at time t has to "remember" the outcome of the last nonempty slot. The LSBFA requires no memory by the users.

We now proceed to compare the two algorithms in terms of robustness in the presence of channel errors. The LSBFA resolves any collisions using the CTMCRA, while the "Stack" algorithm

resolves any collisions using the CMTMCRA. In [14] Massey showed that under the more realistic situation where channel noise can affect the transmissions on the forward and/or feedback channel, the CTMCRA is extremely robust, while the CMTMCRA can suffer deadlock. It is clear that the "Stack" algorithm, even though it uses the CMTMCRA, does not suffer deadlock because of the continuous entry of new packets into the random access system. Nevertheless, we can easily show that it is less robust than the LSBFA. From all the possible types of errors consider the one where an "empty" slot is detected as a "collision" slot by the blocked users because of noise on the channel. In the terminology of "Stack" algorithm, all users with a packet in cell "r" ($r \geq 1$) of the stack place their packets in cell "r+1". Thus, each time an "empty-to-collision" error occurs, both cell "0" and cell "1" become empty, and the algorithm proceeds to resolve a nonexisting collision. All blocked users detect a sequence of empty slots but they do not move their packets downwards, since the last nonempty slot was a collision slot (in this case a false collision). This deadlock situation lasts until some new packet(s) enters the system. This happens with probability $1-p(0)$ independently at each slot. Therefore, the average length of a deadlock period is $l_d = 1/(1-p(0))$ slots. Under light traffic conditions, the average deadlock period becomes long resulting in increased average packet delay. For example, if $p(\cdot)$ is the Poisson distribution with $\lambda = 0.1$, then $l_d \simeq 10.5$ slots.

In contrast to the "Stack" algorithm, the LSBFA overcomes the same error by wasting only two slots independently of the input intensity, as it can be seen from the rules of the algorithm given in section 2.

Using similar arguments like the one used in the "empty-to-collision" type of error case, we can easily show that the LSBFA compared to the "Stack" algorithm exhibits superior robustness in the presence of channel errors of every possible sort.

APPENDIX A

Proof of Theorem 3

Existence Consider the sequence $X^{(n)}$, n=0, 1, 2, . . ., defined as follows:

$$x_0^{(n)} = x_1^{(n)} = 1$$

$$x_k^{(n)} = (A^n)_k[X^{(0)}]; \quad k \geq 2 \tag{A.1}$$

where

$$X^{(0)} = \{x_k^{(0)}\}, \quad x_k^{(0)} = bk-c, \ k \geq 2$$

$b \epsilon R^+$, $c = 2b\lambda+1$, and A is the operator defined in (10).

For n=1, after simple calculations, we obtain

$$x_k^{(1)} = x_k^{(0)} - 2p(0)(2^{-k}f_k b-2^{-k}g_k), \quad k \geq 2 \tag{A.2}$$

where

$$f_k(\lambda, r) = (1-2\lambda)k + r(1-2\lambda) - 2\lambda$$

$$g_k(r) = 2(k+r+1)$$

$$\lambda = \sum_{i=0}^{\infty} ip(i), \ r = p(1)/p(0)$$

From (A.1) and (A.2] for $k \geq 2$, we have

$$x_k^{(n+1} = x_k^{(n)} - 2p(0)((B^n)_k[F(\lambda, r)]b - (B^n)_k[G(r)]) \tag{A.3}$$

where

$$F(\lambda, r) = \{2^{-k}f_k(\lambda, r\}$$

$$G(r) = \{2^{-k}g_k(r)\}$$

and the operator B is as defined in (11).

If the p.m.f. of the input increment $p(\cdot)$ satisfies condition (13), then

$$(B^{n_0})_k[F(\lambda, r)] > 0 \text{ for some } n_0 < \infty \text{ and every } k \geq 2$$

Hence, in view of (A.3) we can always find a $b \in R^+$ such that

$$x_k^{(n+1)} \leq x_k^{(n)} \quad \text{for every } n \geq n_0 \text{ and } k.$$

For a given $p(\cdot)$, let

$$b = \max_k((B^{n_0})_k[G(r)]/(B^{n_0})_k[F(\lambda, r)]), \quad k \geq 2 \tag{A.4}$$

Clearly, if b is chosen as in (A.4), then for every $k \geq 2$, the values $x_k^{(n_0)}$, $x_k^{(n_0+1)}$, $x_k^{(n_0+2)}$, . . . form a nonincreasing sequence.

If $\lambda < 1/2$, then $x_k^{(n)}$ is nonnegative for every k and n, since with b chosen as in (A.4) $x_k^{(0)}$ is nonnegative for every k and the operator A is nonnegative ($q_k(\cdot) \geq 0$). Note that if $\lambda \geq 1/2$, then there is no n_0 for which (13) is true.

Since $x_k^{(n_0)}$, $x_k^{(n_0+1)}$, $x_k^{(n_0+2)}$, . . . is a nonnegative nonincreasing sequence the following limit exists for every $k \geq 2$:

$$x_k = \lim_{n\to\infty} x_k^{(n)} \tag{A.5}$$

Clearly, if (13) is true, then $0 \leq x_k^{(n)} < \infty$ for every $n \leq n_0$ and $2 \leq k < \infty$. Moreover for every $n \geq n_0$ we have

$$x_k^{(n+1)} \triangleq 1 + 2\sum q_k(m)\, x_m^{(n)} \leq x_k^{(n_0)} < \infty$$

Thus, for every n and $2 \leq k < \infty$ the nonnegative series

$$\sum_{m=0}^{\infty} q_k(m)\, x_m^{(n)}$$

is absolutely convergent (satisfies condition (12)). Therefore, for arbitrary $\varepsilon > 0$ and n there exists a K_0, such that for arbitrary $K > k_0$ we have:

$$\sum_{m=K}^{\infty} q_{k(m)} x_m^{(n)} < \varepsilon/10, \quad k < K \tag{A.6}$$

In view of (A.5) and for the given ε and K we have

$$|x_k^{(n)} - x_k| = x_k^{(n)} - x_k < \varepsilon/5, \text{ for all } n > N \geq n_0 \text{ and } k < K \tag{A.7}$$

Then, for arbitrary $k_0 < \infty$ we obtain

$$|x_{k_0} - A_{k_0}[X]| \leq |x_{k_0} - 2 \sum_{m=0}^{K-1} q_{k_0}(m) x_m - 1| + \frac{\varepsilon}{5} \leq$$

$$|x_{k_0}^{(n+1)} - 2 \sum_{m=0}^{K-1} q_{k_0}(m)\; x_m^{(n)} - 1| + \frac{4}{5}\varepsilon \leq |x_{k_0}^{(n+1)} - A_{k_0}[X^{(n)}]| + \varepsilon = \varepsilon$$

Hence $X = \{x_k\}$ is a solution of system (9), since the above inequality is true for arbitrarily small $\varepsilon > 0$.

We proceed now with the lower bound sequence ${}^{(n)}X$, n = 0, 1, 2, . . which is defined as follows:

$${}^{(n)}x_0 = {}^{(n)}x_1 = 1$$

$${}^{(n)}x_k = (A^n)_k[{}^{(0)}X], \quad k \geq 2$$

where

$${}^{(0)}X = \{{}^{(0)}x_k\}, \quad {}^{(0)}x_k = b'k-c', \; k \geq 2$$

$$b' = 2/(1-2\lambda), \; c' = 2b'\lambda + 1$$

for n=1 after simple calculations, we obtain

$${}^{(1)}x_k = {}^{(0)}\,x_k - 2p(0)2^{-k}b' \tag{A.8}$$

In view of the fact that the operator A is nonnegative ($q_k(\cdot) \geq 0$), it follows from (A.8) that for $\lambda < 1/2$ and for every $k \geq 2$, the values ${}^{(0)}x_k$, ${}^{(1)}x_k$, ${}^{(2)}x_k$, . . . form a nondecreasing sequence. This sequence is also bounded from above by the sequence $X^{(n)}$. Indeed, if condition (13) is true, then $b' < b$, where b is as defined in (A.4). (If $b' \geq b$ then it follows from (A.2) that $x_k^{(1)} \geq x_k^{(0)}$ for every $k \geq 0$ and consequently condition (13) cannot be true.) Thus, ${}^{(0)}x_k = b'k-c' < bk-c = x_k^{(0)}$ for every $k \geq 2$ and since the operator A is nonnegative it follows that

$$ {}^{(n)}x_k \leq x_k^{(n)} \quad \text{for every } n \text{ and } k \geq 2 . $$

Since ${}^{(0)}x_k$, ${}^{(1)}x_k$, ${}^{(2)}x_k$, . . . is a nondecreasing sequence, which is bounded from above by the sequence $x^{(n)}$, the following limit exists for every $k \geq 2$:

$$ x_k' = \lim_{n\to\infty} {}^{(n)}x_k \leq x_k \tag{A.9} $$

Following the same steps as in the case of $X = \{x_k\}$ we can prove that the sequence $X' = \{x_k'\}$ is a solution of system (9) satisfying (12).

<u>Uniqueness</u> To prove the uniqueness of a solution we assume that system (9) has two distinct solutions satisfying (12) and we end up with a contradiction. The proof parallels the one given in [12, p. 236] and it is omitted.

From the uniqueness of the solution, we have

$$ x_k' = \lim_{n\to\infty} {}^{(n)}x_k = \lim_{n\to\infty} x_k^{(n)} = x_k $$

APPENDIX B

Proof of Lemma 1

We prove lemma 1 by induction. For n=1, we have

$$\frac{1}{2}\, B_k[Y] \triangleq \sum_{m=2} q_k(m)(a_0 + b_0 m) v_i^m$$

$$= a_0 \sum_{m=0}^{\infty} q_k(m) v_0^m + b_0 \sum_{m=0}^{\infty} q_k(m) m v_0^m - (a_0 q_k(0) + \frac{1}{2}(a_0 + b_0) q_k(1) \tag{B.1}$$

where $q_k(\cdot)$ is as defined in (8) and $v_0 = 0.5$. In particular, $q_k(0) = \exp(-\lambda) v_0^k$, and $q_k(1) = \exp(-\lambda)(k+\lambda) v_0^k$. After simple calculations, we obtain

$$\sum_{m=0}^{\infty} q_k(m) v^m = \exp(-\lambda(1-v))\left(\frac{1+v}{2}\right)^k \tag{B.2}$$

$$\sum_{m=0}^{\infty} q_k(m) m v^m = \exp(-\lambda(1-v))\left(\frac{v}{1+v}\; k+\lambda v\right)\left(\frac{1+v}{2}\right)^k \tag{B.3}$$

Substitution of $q_k(0)$, $q_k(1)$, (B.2), and (B.3) with $v=v_0$ into (B.1) gives

$$B_k[Y] = a_1(0) v_0^k + a_1(1) v_1^k + b_1(0) k v_0^k + b_1(1) k v_1^k \tag{B.4}$$

where

$$v_1 = (1+v_0)/2,$$

$$a_1(0) = -2\exp(-\lambda)(a_0 + \lambda v_0(a_0+b_0)),$$

$$b_1(0) = -2\exp(-\lambda) v_0 (a_0+b_0),$$

$$a_1(1) = 2\exp(-\lambda(1-v_0))(a_0+\lambda v_0 b_0),$$

$$b_1(1) = 2\exp(-\lambda(1-v_0) v_0 (1+v_0)^{-1} b_0$$

Equation (B.4) proves the lemma for n=1.

Next we assume that (16) is true for n=j and we prove that it is true for n=j+1 as well. We have

$$(B^j)_k[Y] = \sum_{i=0}^{j} (a_j(i) + kb_j(i))v_i^k$$

$$(B^{j+1})_k[Y] = B_k[B^j[Y]] = 2\sum_{m=2}^{\infty} q_k(m) \sum_{i=0}^{j} (a_j(i) + mb_j(i))v_i^m$$

$$= 2(\sum_{i=0}^{j} a_j(i) \sum_{m=0}^{\infty} q_k(m) v_i^m + \sum_{i=0}^{j} b_j(i) \sum_{m=0}^{\infty} q_k(m) m v_i^m -$$

$$- (q_k(0) \sum_{i=0}^{j} a_j(i) + q_k(1) \sum_{i=0}^{j} (a_j(i) + b_j(i)v_i)) \tag{B.5}$$

Substitution of $q_k(0)$, $q_k(1)$, (B.2) and (B.3) with $v=v_i$ into (B.5) gives

$$(B^{j+1})_k[Y] = \sum_{1=0}^{j+1} (a_{j+1}(m) + kb_{j+1}(m))v_m^k \tag{B.6}$$

where $a_{j+1}(m)$, $b_{j+1}(m)$, m=0, 1, . . ., j+1, are as given in (16). Equation (B.6) proves the lemma for n=j+1.

APPENDIX C

Proof of Theorem 5

Consider the following truncated system of equations

$$x_k = 1 + 2 \sum_{m=0}^{n} q_k(m) x_m, \qquad 2 \le k \le n$$

or in matrix notation

$$(\tfrac{1}{2} I_n - Q_n)\underline{x}_n = \underline{c}_n \tag{C.1}$$

where

$$\underline{x}_n = (x_2, x_3, \ldots, x_{n+1})^T$$

$$\underline{c}_n = (c_2, c_3, \ldots, c_{n+1})^T, \; c_k = \tfrac{1}{2} + q_k(0) + q_k(1),$$

$$k = 2, 3, \ldots, n+1$$

$Q_n = (q_{ij})$ is a (nxn) nonnegative, irreducible, square matrix with $q_{ij} = q_i(j)$, $2 \le i \le n+1$, $2 \le j \le n+1$, and I_n is the (nxn) unit matrix

The following theorem gives the conditions that ensure positivity ($\underline{x} > 0$) of solutions to the equation system (C.1).

Theorem [16, Th. 2.1]

A necessary and sufficient condition for a solution $\underline{x}_n (\underline{x}_n \ge \underline{0}, \ne \underline{0})$ to equations (C.1) to exist for any $\underline{c}_n \ge \underline{0}, \ne \underline{0}$ is that $r_n < 1/2$; where r_n is the Perron-Frobenius eigenvalue of the nonnegative, irreducible matrix Q_n. In this case, there is only one solution $\underline{x}_n$, which is strictly positive and given by

$$\underline{x}_n = (\tfrac{1}{2} I_n - Q_n)^{-1} \underline{c}_n$$

It is known from the theory of the nonnegative matrices (see [16]) that

$$r_{n+1} > r_n, \quad \text{and} \lim_{n\to\infty} r_n = r \qquad \text{(C.2)}$$

where r is the Perron-Frobenius eigenvalue (the reciprocal "convergence norm") of the infinite dimensional matrix Q; where $Q=\lim Q_n$ as $n\to\infty$ (Q_n is the (nxn) northwest corner truncation of Q).

From the above theorem and (C.2) it follows that if for some n_0 we have $r_{n_0} > 1/2$ then for all $n \geq n_0$ system (C.1) has no nonnegative solution. In this case, equation system (9) has no nonegative solution satisfying (12), since it is obtained from equation system (C.1) as $n\to\infty$. To calculate the Perron-Frobenius eigenvalue r_n we made use of the following lemma:

<u>Lemma</u> [17]

If s(T) and S(T) are the minimal and maximal row sums of a square nonnegative, irreducible matrix T with Perron-Frobenius eigenvalue r, then

$$s(T) \leq (s(T^2))^{1/2} \leq (s(T^4))^{1/4} \leq \ldots \leq r \leq \ldots \leq$$
$$\leq (S(T^4))^{1/4} \leq (S(T^2))^{1/2} \leq S(T)$$

Calculations made for $n_0 = 10$ show that $r_{10} > 1/2$ for $\lambda = 0.363$. Thus, for $\lambda \geq 0.363$ equation system (9) has no nonnegative solution satisfying (12).

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k	$L_k(0)$	$L_k(.05)$	$L_k(.10)$	$L_k(.15)$	$L_k(.20)$	$L_k(.25)$	$L_k(.30)$
2	5.0000	5.6196	6.4780	7.7456	9.8057	13.7369	24.1140
3	7.6666	8.7282	10.1977	12.3662	15.8884	22.6068	40.3291
4	10.5238	12.0455	14.1520	17.2608	22.3103	31.9422	57.3431
5	13.4190	15.4054	18.1553	22.2137	28.8060	41.3809	74.5359
6	16.3130	18.7646	22.1586	27.1674	35.3036	50.8234	91.7358
7	19.2009	22.1173	26.1548	32.1133	41.7919	60.2535	108.9139
8	22.0853	25.4663	30.1468	37.0542	48.2741	69.6756	126.0768
9	24.9690	38.8144	34.1377	41.9938	54.7546	79.0950	143.2339
10	27.8532	32.1629	38.1291	46.9339	61.2355	88.5150	160.3906

Table 1. $L_k(\lambda)$: Mean length of a session of multiplicity k.

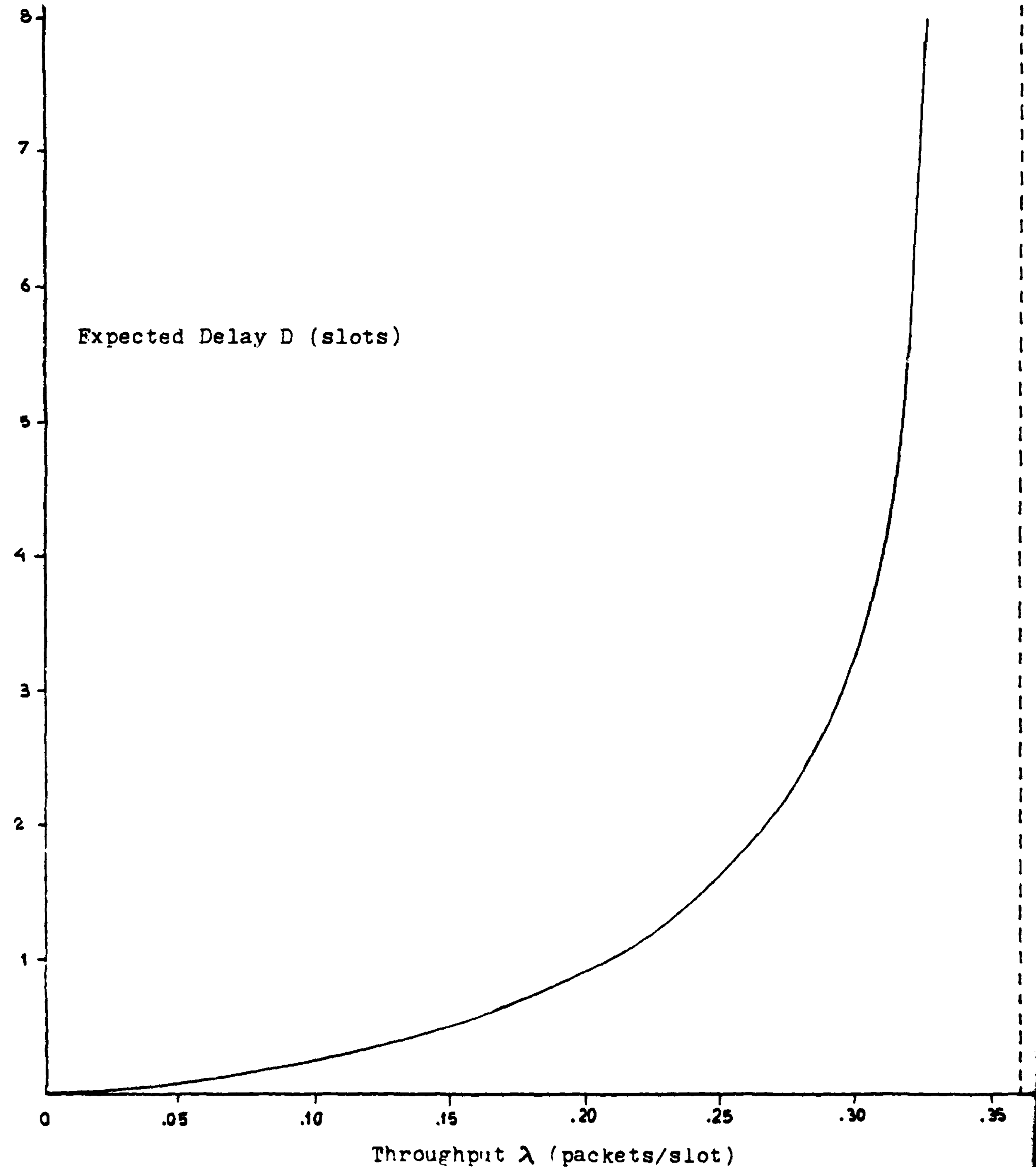

Fig. 1. The upper bound L-1 on the expected packet delay D for the LSBFA.

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